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Intercommutation of $U(1)$ global cosmic strings

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ABSTRACT: Global strings (those which couple to Goldstone modes) may play a role in cosmology. In particular, if the QCD axion exists, axionic strings may control the efficiency of axionic dark matter abundance. The string network dynamics depend on the string intercommutation efficiency (whether strings re-connect when they cross). We point out that the velocity and angle in a collision between global strings “renormalize” between the network scale and the microscopic scale, and that this plays a significant role in their intercommutation dynamics. We also point out a subtlety in treating intercommutation of very nearly antiparallel strings numerically. We find that the global strings of a $O(2)$ -breaking scalar theory do intercommute for all physically relevant angles and velocities.

KEYWORDS: axions, dark matter, cosmic strings, global strings

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1 Introduction

{sec:intro}

Cosmic strings [1, 2] are hypothetical extended solitonic excitations which may play a significant role in cosmology. Their original motivation, for structure formation [3–5], appears in conflict with modern microwave sky data [6]. But cosmic strings may be important in other contexts. In particular, if the QCD axion [7, 8] exists, the axion field may contain a string network in the early Universe [9] which may dominate axion production and play a central role in the axion as a dark matter candidate.

String defects can occur whenever the vacuum spontaneously breaks a symmetry, say, breaking G down to $H \in G$, such that the quotient group (vacuum manifold) G/H has nontrivial π_1 homotopy. The simplest example is the complete breaking of an $SO(2)$ or $U(1)$ symmetry. Generally, strings are divided into two sorts; “local” strings, which typically occur when G is a gauge group and which do not couple to any massless fields [1], and “global” strings, which typically occur when G is a global (non-gauged) symmetry, in which case the strings couple to the associated massless Goldstone bosons [10]. There is a rich literature studying local strings and their networks [11–13]. This includes a rather careful study of when crossing strings intercommute with each other [14–17].

Global strings, such as the axionic string networks alluded to above, are harder to study. The strings interact with each other via massless fields, which change the network’s dynamics, both by allowing strings to radiate energy efficiently [18–21], and by communicating inter-string forces. To simulate such a network faithfully requires a hybrid algorithm, which treats the string cores via a Nambu-Goto action and treats the Goldstone field with lattice methods [22], together with some string-field interaction. We have recently presented such an algorithm [23]. However, to implement it we must know when strings which cross each other will inter-commute, and when they will simply pass through each other (see Figure 1). This problem has been previously addressed via microscopic simulations [24]. But we will argue here that the physics is more complicated due to the long range inter-string interactions, and the issue of string inter-commutation deserves some more study.

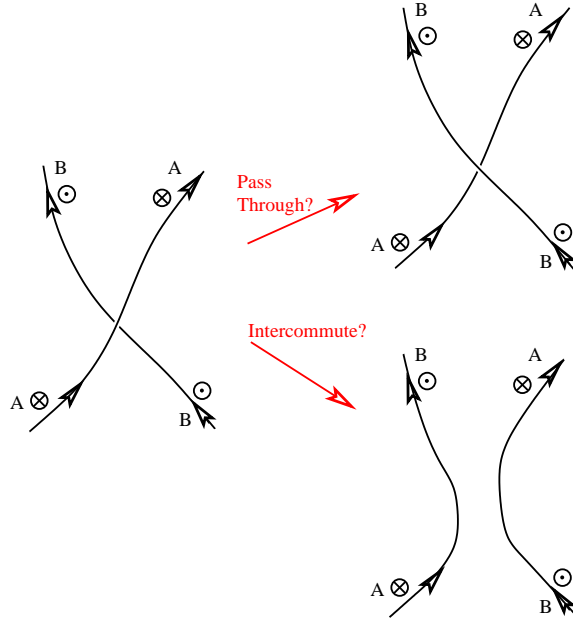


Figure 1. ^{fig:intercommute} Left: two strings approach, with A descending and B rising out of the page. The arrows indicate the “sense” of the string. There are two possibilities: they pass through each other (top right), or they intercommute (bottom right). Which occurs is determined by the microphysics at the point where they touch.

Cosmic string networks involve two disparate scales. There is a microscopic scale m_h^{-1} , set by the inverse mass of the (Higgs) radial excitation of the symmetry-breaking field. And there is a macroscopic scale, the average inter-string separation L , which is typically of order the Hubble scale H^{-1} . For axions around the time where the dark matter density is established, these scales differ by a factor of $\sim 10^{30}$ [25]. The energy in a local cosmic string is carried within a few m_h^{-1} of the string’s core. But for a global string, the energy is distributed logarithmically over all scales between m_h^{-1} and L . As global strings approach each other through these intermediate scales, this energy distribution is responsible for inter-string interactions, which apply both torques, and attraction or repulsion, to the strings. The angle and velocity with which the strings approach therefore evolves logarithmically with scale. We illustrate this idea in Figure 2. Here we study this evolution, and its impact on the physics of string intercommutation. We concentrate on strings arising from a complex scalar (the $O(2)$ model or relativistic 3D xy model); the details and results could be quite different for strings with other more complicated symmetry breaking patterns.

In the next section we review the physics of global cosmic strings, and we explain why long range inter-string interactions can be important. Section 3 treats the inter-string forces explicitly and finds how the angle and velocity of string approach changes with scale. This is combined with a new study of microscopic string intercommutation in Section 4.

We end with a discussion and conclusion, but we will give the main findings here. At the macroscopic scale strings may approach each other at a range of angles and relative

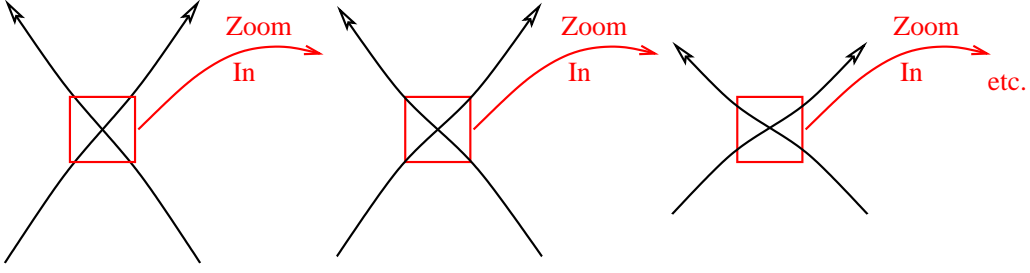


Figure 2. ^{fig:scales} Two string just before crossing. They twist towards larger crossing angle; zooming in, we see that at short distances they twist more; zooming in, they twist still more, and so on. Even if the twist is slight, over many orders of magnitude in scale it can become very large.

velocities. But accounting for inter-string forces, at the microscopic scale of $100/m$, the strings are essentially always almost antiparallel (the attractive channel) and approaching with a highly relativistic velocity $v \simeq 0.9$. We show in contradiction to previous results [24] that such strings always intercommute. Therefore in treating O(2)-model global strings, one should assume that intercommutation always occurs.

2 Global string review

Here we review global strings (see also [2, 10, 26]). Consider a complex scalar field φ with U(1) symmetry (or equivalently, two real scalars φ_r, φ_i with an O(2) symmetry; $\varphi = (\varphi_r + i\varphi_i)/\sqrt{2}$). The most general renormalizable Lagrangian is¹

$$-\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi + \frac{\lambda}{8} (2\varphi^* \varphi - f_a^2)^2. \quad (2.1) \quad \{\text{Lagrangian}\}$$

If $f_a^2 > 0$ then the classical vacuum $\sqrt{2}\varphi = f_a e^{i\theta_A}$ spontaneously breaks the U(1) symmetry. The excitation spectrum about this vacuum contains a radial “Higgs” excitation with mass $m_h^2 = \lambda f_a^2$ and angular excitations, which are massless Goldstone modes. We are interested in solitonic solutions corresponding to an extremely large number of quanta, so a classical description of the field and its dynamics are sufficient.

Now θ_A is only defined modulo 2π , and there are configurations where it cannot be defined in a continuous and single-valued way. Specifically, a topological string solution is where there is a 1-dimensional oriented (linelike) locus of points, called a cosmic string, where $|\varphi| = 0$, and such that θ_A varies by 2π as one circles the string with positive sense. Locally the string is nearly straight and we can use it as the z -axis in polar (z, r, ϕ) coordinates. The string solution is $\varphi(z, r, \phi) = f_a h(r) \exp(i(\phi - \phi_0))$, with $h(r)$ chosen to minimize the energy per unit length (string tension)

$$\begin{aligned} T &= \int r dr d\phi (|\nabla \varphi|^2 + V(\varphi^* \varphi)) \\ &= \pi f_a^2 \int r dr \left((\partial_r h)^2 + \frac{h^2}{r^2} + \frac{m_h^2}{8} (h^2 - 1)^2 \right), \end{aligned} \quad (2.2) \quad \{\text{E}_h\}$$

¹We use the $[-, +, +, +]$ or mostly-positive metric

which is minimized when $h(r)$ obeys

$$\partial_r^2 h + \frac{1}{r} \partial_r h + \frac{h^2}{r^2} - \frac{m_h^2}{2} h(h^2 - 1) = 0, \quad h(0) = 0, \quad \lim_{r \rightarrow \infty} h(r) = 1. \quad (2.3) \quad \{\text{h_of_r}\}$$

The most important term in Eq. (2.2) is the $\pi f_a^2 \int r dr h^2/r^2$ term, arising from $|\nabla_\phi \varphi|^2$. All other terms only contribute appreciably for $m_h r \sim 1$ and fall off quickly at large r . But this term receives equal contributions from all logarithmic scales larger than m_h^{-1} :

$$T \simeq \int_{m_h^{-1}}^{r_{\max}} r dr \pi f_a^2 (1/r^2) = \pi f_a^2 \left(\ln(m_h r_{\max}) + \mathcal{O}(1) \right), \quad (2.4) \quad \{\text{Tension}\}$$

where r_{\max} is a long-distance cutoff, which would physically be provided by the curvature scale of the string or the distance to the next string. This logarithmic energy scaling will be essential to our arguments.

The other essential feature of the strings is the way they interact with their environment. Suppose that a string exists in an environment where θ_A also varies uniformly, for instance because of the far field of another string. Outside the string core the solution is described by θ_A alone, and its equation of motion is linear so we can superpose solutions. In this case our *Ansatz* for the field becomes

$$\varphi(x) = f_a h(r) e^{i\phi} e^{i\theta_{A,\text{ext}}}, \quad \theta_{A,\text{ext}} \simeq x_i \nabla_i \theta_{A,\text{ext}}, \quad (2.5) \quad \{\text{Ansatz}\}$$

where we have approximated the external field by the first term in its Taylor series. Near the string core the equation of motion is not linear, so $\nabla_i \theta_{A,\text{ext}}$ will cause an acceleration in the string. The easy way to determine this is to find the force per unit length on the string, by integrating the stress normal to a boundary around the string which we draw at some radius $r \gg m_h^{-1}$:

$$dF_i = \int r d\phi T_{ij}(r, \phi) \hat{n}_j. \quad (2.6) \quad \{\text{dF}\}$$

Here T_{ij} is the stress tensor

$$T_{ij} = f_a^2 \left(\partial_i \theta_A \partial_j \theta_A - \frac{1}{2} \delta_{ij} \partial_k \theta_A \partial_k \theta_A \right). \quad (2.7) \quad \{\text{Tij}\}$$

Now $\partial_i \theta_A = \partial_i \theta_{A,\text{ext}} + \hat{\phi}_i/r$. The $(\theta_{A,\text{ext}})^2$ term and the $(\hat{\phi})^2$ term each integrate to zero, but the cross term contributes

$$dF_i = 2\pi f_a^2 \epsilon_{ijz} \nabla_j \theta_{A,\text{ext}}. \quad (2.8) \quad \{\text{dFis1}\}$$

Now z appears because it is the unit tangent of the string; so this generalizes for a string of unit tangent \hat{s} to

$$dF_i = 2\pi f_a^2 \epsilon_{ijk} \nabla_j \theta_{A,\text{ext}} s_k, \quad (2.9) \quad \{\text{dFis2}\}$$

or for a moving string with 4-velocity $v^\mu = (1, \vec{v})$, to

$$dF_\mu = 2\pi f_a^2 \epsilon_{\mu\nu\alpha\beta} \nabla^\nu \theta_{A,\text{ext}} s^\alpha v^\beta. \quad (2.10) \quad \{\text{dFis3}\}$$

These are the facts about strings that we will need in what follows.

3 Renormalization of string angle and velocity

{sec:renorm}

Consider two strings, approaching each other and at some relative angle. When the string separation z is small compared to the characteristic inter-string spacing L , the strings can be taken as nearly straight and moving with nearly uniform velocity. Then we can work in the frame where one string stretches in the y direction and the other in the xy plane, and they approach each other along the z direction with equal and opposite velocities v . The system is fully specified by the velocity v , the separation z (or equivalently the time to impact $t = z/2v$ with the 2 because each string is moving), and the angle ϕ , describing how far from parallel the two strings are.

The strings exert forces on each other. The z component of the force is repulsive if $\phi < \pi/2$ (nearly parallel) and attractive if $\phi > \pi/2$ (nearly antiparallel). There are also forces in the xy plane, which tend to push the strings into the antiparallel relative orientation. We want to understand how these forces change the relative velocity and orientation of the strings, as the inter-string distance drops from the macroscopic to the microscopic scale.

The characteristic size of this force is $dF \sim 2\pi f_a^2/z$, while the tension of the string, which sets its inertia, is $T = \pi f_a^2 \ln(zm_h)$. Therefore, in the time $t \sim z/2v$ which it takes for the strings to get a factor of 2 closer together, the velocity and orientation of the strings can change by an amount $t dF/T \sim 1/[v \ln(zm_h)]$. This is small by one power of our large logarithm. But the number of factors-of-two over which the strings must approach each other is large, $\sim \ln(zm_h)$. Therefore, even if a factor-of-2 change in the separation only makes a small correction to the relative velocity and angle near the intersection point, the strings' relative velocity and angle of approach will change significantly as the separation goes from the macroscopic to the microscopic scale. This is analogous to renormalization group flow. At each scale, it is the v, ϕ value at that scale which is relevant. Over a factor of 2 change in scale, these change by a small amount, but the cumulative change can be large if the log of the ratio of scales is large enough. In this section we will find a differential equation for how v, ϕ evolve with the log of the separation scale.

First we estimate of whether the change will be large. Introducing $\kappa = \ln(zm_h)$, the total change in velocity and angle is of order

$$\Delta(v, \phi) \sim \int_{\kappa_{\min}}^{\kappa_{\max}} \frac{d\kappa}{\kappa} \sim \ln \frac{\kappa_{\max}}{\kappa_{\min}}. \quad (3.1) \quad \{\text{handwave}\}$$

For axionic strings at a temperature around 1 GeV we have $\kappa_{\max} \sim \ln(f_a/H) \sim \ln(10^{30}) \sim 70$. The short distance physics can be studied with lattice methods, but only using $\kappa_{\min} \sim \ln(N_{\text{sites}}) \sim \ln(10^2) \sim 5$. Then $\ln(\kappa_{\max}/\kappa_{\min}) \simeq \ln(70/5) \simeq 2.6$ is actually fairly large.

Let us move forward to a quantitative calculation of how the strings' relative angle and velocity change with scale. We will assume that the string is *nearly* straight, and treat both the string's curvature and the force on the string to be $\mathcal{O}(\kappa^{-1})$. We work to order κ^{-1} , which means we may treat the string as straight and its motion as uniform in computing the force per unit length on each string. First we compute this force. For the string on the y axis at the moment when its z -coordinate is z_0 , in its rest frame we have

$\theta_A = \text{atan}(x/(z - z_0))$. So in the frame where it moves in the $+z$ direction with velocity v and at the moment t_0 when the string is at z_0 , we have

$$\theta_A = \text{atan} \frac{x}{\gamma([z - z_0] - v[t - t_0])}, \quad \partial_x \theta_A = \frac{\gamma[z - z_0]}{x^2 + \gamma^2[z - z_0]^2}, \quad \partial_z \theta_A = \frac{-\gamma x}{x^2 + \gamma^2[z - z_0]^2}, \quad \partial_t \theta_A = -v \partial_z \theta_A. \quad (3.2) \quad \{\text{gradients}\}$$

Here $\gamma = 1/\sqrt{1 - v^2}$ as usual.

We take the point in the (x, y) plane where the strings will cross to be $(x, y) = (0, 0)$. In the approximation that the upper string is straight, the position varies with the length ℓ along the string from this point as: $z - z_0 \equiv z = 2vt$ and $(x, y) = \ell(\sin \phi, \cos \phi)$. At this point, the force on the upper string, using Eq. (2.10) and Eq. (3.2), is

$$\begin{aligned} dF_\mu &= 2\pi f_a^2 \epsilon_{\mu\nu\alpha\beta} v^\nu \nabla^\alpha \theta_A s^\beta, \\ dF_\perp &= 2\pi f_a^2 \frac{\gamma(1 + v^2)\ell \sin \phi}{\gamma^2 z^2 + \ell^2 \sin^2 \phi}, \\ dF_z &= 2\pi f_a^2 \frac{\gamma z \cos \phi}{\gamma^2 z^2 + \ell^2 \sin^2 \phi}, \\ dF_0 &= -2\pi f_a^2 \frac{\gamma v z \cos \phi}{\gamma^2 z^2 + \ell^2 \sin^2 \phi}. \end{aligned} \quad (3.3) \quad \{\text{Fresult}\}$$

Here dF_0 is the energy exchange rate, and the relative sign between dF_z and dF_0 is because the upper string moves in the $-z$ direction.

Beyond lowest order, the string is not straight. Write the string separation a time t before the strings meet as

$$z(\ell, t) = z_1(t) + t\bar{z}_2(\ell/t), \quad (3.4) \quad \{\text{zof t}\}$$

where $\partial_t z_1 = -2v$ with v the velocity at the crossing point. Here \bar{z}_2 parameterizes how the velocity varies along the string, which should be a function of ℓ/t only as the appearance should be self-similar as we vary scale. Both \bar{z}_2 and $t\partial_t v$ are first-order small. Similarly, rotating the (x, y) plane so the strings are at angles of $\pm\phi/2$ with respect to the y axis, we have

$$x(\ell, t) = \int_0^\ell d\ell' \sin(\phi(\ell', t)/2), \quad \phi(\ell, t) = \phi_1(t) + \phi_2(\ell/t). \quad (3.5) \quad \{\text{phiof t}\}$$

Note that we are defining $\phi(\ell, t)$ in terms of the unit tangent of the string. We assume that $t\partial_t \phi_1$ and ϕ_2 are first-order small, and ϕ_2 is a self-similar function of ℓ/t only. We also introduce the notation $\xi = \ell/t$. We will write $\bar{z}_2' = \partial_\xi \bar{z}_2(\xi)$, so $\partial_\ell \bar{z}_2 = t^{-1} \bar{z}_2'$ and $\partial_t \bar{z}_2 = -\xi t^{-1} \bar{z}_2'$, and similarly for $\phi_2(\xi)$.

Introducing the energy per unit length of the string,

$$\varepsilon = \pi \kappa f_a^2 \gamma = \gamma T, \quad (3.6) \quad \{\text{defeps}\}$$

the equation of motion for the string is [27]

$$\partial_t^2 x_i = (1 - v^2) \partial_\ell^2 x_i + \varepsilon^{-1} (F_i - F_0 \partial_t x_i). \quad (3.7) \quad \{\text{stringEOM}\}$$

Using that $F_0 = v_i F_i$, the last term can be rewritten as $\varepsilon^{-1} (\delta_{ij} - v_i v_j) F_j$, where ε^{-1} accounts for the string's inertia per unit length, and $(\delta_{ij} - v_i v_j)$ is the usual relativistic reduction of the acceleration for a force acting along the current direction of propagation.

Consider first the z -motion. We have

$$\begin{aligned}\partial_t z &= -2v + \bar{z}_2 - \xi \bar{z}_2', \\ \partial_t^2 z &= -2\partial_t v + \frac{\xi^2}{t} \bar{z}_2'', \\ \partial_\ell z &= \bar{z}_2', \\ \partial_\ell^2 z &= \frac{1}{t} \bar{z}_2''.\end{aligned}\tag{3.8} \quad \{\text{zmotion1}\}$$

Inserting these into Eq. (3.7) and using Eq. (3.3), we find

$$-\partial_t v + \frac{\xi^2 - (1 - v^2)}{2t} \bar{z}_2'' = \frac{1 - v^2}{\pi \kappa f_a^2 \gamma} F_z = \frac{2}{\kappa} \frac{(1 - v^2) z \cos \phi}{z^2 / (1 - v^2) + \ell^2 \sin^2 \phi}.\tag{3.9} \quad \{\text{zmotion2}\}$$

The factor of 2 difference between Eq. (3.8) and the left-hand side of Eq. (3.9) is because each string feels a force, or equivalently because each string must only move a distance $z/2$ before they meet, so Eq. (3.7) should be applied to $z/2$ not z .

This expression is a complicated differential equation for both $v(t)$ and for $\bar{z}_2(\xi)$. However, at the special point $\xi^2 = (1 - v^2)$, the \bar{z}_2'' term drops out and we directly find $\partial_t v$. Evaluating at this point, and using $z = 2vt$ on the right hand side, we find

$$\frac{t dv}{dt} = -\frac{2}{\kappa} \frac{2(1 - v^2)^2 v \cos \phi}{4v^2 + (1 - v^2)^2 \sin^2 \phi}.\tag{3.10} \quad \{\text{zmotion3}\}$$

The relative velocity rises if $\phi > \pi/2$ and it falls if $\phi < \pi/2$, as expected.

The reason that the \bar{z}_2 terms drop out at $\xi^2 = 1 - v^2$ is because waves propagate along the string with velocity 1, which in the xy plane is velocity $\sqrt{1 - v^2}$. The point $\xi^2 = 1 - v^2$ is the point where a wave will reach $\ell = 0$ at the moment the strings meet. Larger ξ values cannot communicate with the intersection point, and any dynamics at smaller ξ is carried, as a wave, past the intersection point before the strings collide.

Now we repeat the analysis for the in-plane (angular) evolution. Starting with Eq. (3.5), we find

$$\begin{aligned}\frac{d^2 x}{d\ell^2} &= \frac{\cos(\phi/2)}{2t} \phi_2', \\ \frac{d^2 x}{dt^2} &= \frac{\xi t \cos(\phi/2)}{2} \partial_t^2 \phi_1 + \frac{\xi^2 \cos(\phi/2)}{2t} \phi_2'.\end{aligned}\tag{3.11} \quad \{\text{transmotion1}\}$$

The expressions for y are the same but with $\sin(\phi/2)$. Therefore the transverse motion of the string is

$$\frac{\xi^2 - (1 - v^2)}{2t} \phi_2' + \frac{\xi t}{2} \partial_t^2 \phi_1 = \frac{2}{\kappa} \frac{(1 + v^2) \ell \sin \phi}{z^2 / (1 - v^2) + \ell^2 \sin^2 \phi}.\tag{3.12} \quad \{\text{transmotion2}\}$$

The factor of $1/2$ appearing in each term on the left-hand side arises because each string changes orientation. Again this is a differential equation for both $\phi_1(t)$ and for $\phi_2(\xi)$, but ϕ_2 drops out at the special point $\xi^2 = 1 - v^2$ for the same physical reason as before. Evaluating at this point, we find

$$t^2 \partial_t^2 \phi_1 = \frac{2}{\kappa} \frac{2(1 + v^2)(1 - v^2) \sin \phi}{4v^2 + (1 - v^2)^2 \sin^2 \phi}.\tag{3.13} \quad \{\text{transmotion3}\}$$

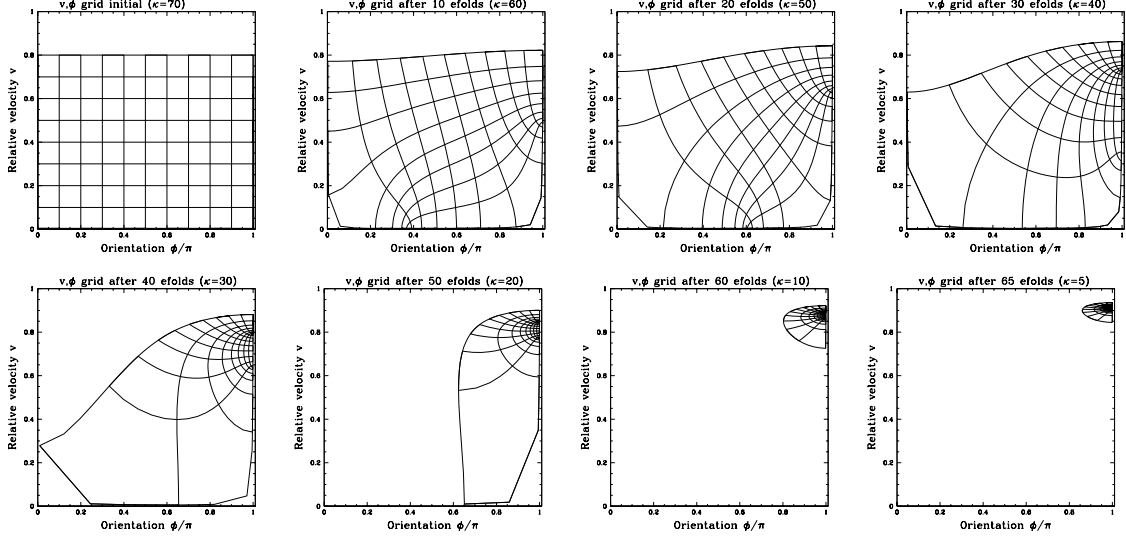


Figure 3. ^{fig:grid} Evolution of ϕ and v (x and y axes) with logarithmic scale, $\kappa \equiv \ln(zm_h)$. Starting with a grid over (ϕ, v) space at large separation ($\kappa = 70$, our axion-motivated choice for an initial scale hierarchy) in the top left, we see how the initial (ϕ, v) value evolves in steps of 10 in κ or e^{-10} in scale.

The most general solution is $\phi_1 = A \ln(t_0/t) + B + Ct$, where A is the right hand side. The term C rapidly becomes subdominant as we go to small times – it is “renormalization group irrelevant” – and we should ignore it. Then $B = \phi(t_0)$, and the coefficient A tells how the angle scales logarithmically with time – or equivalently with scale. The angle between the strings increases for any value of $\phi < \pi$, so the strings always become more antiparallel. This is as expected; objects typically rotate to be in the attractive channel.

Eq. (3.10) and Eq. (3.13) together give us our “renormalization group” equations for understanding how the relative velocity and angle of the strings change with scale. Note that the denominator becomes singular if $v \rightarrow 0$ and $\sin(\phi) \rightarrow 0$ at the same time. This is because slow-moving strings are close together and therefore exert larger forces on each other; and the physically relevant point, $\ell = t/\gamma$ ($\xi^2 = (1 - v^2)$) is at a small separation if ϕ is also close to 0 or π . For all other cases the equations predict smooth evolution with scale.

Figure 3 shows how (ϕ, v) evolve with scale. To see the final value of (ϕ, v) for some initial choice, find the grid position corresponding to the initial choice in the upper left frame of the figure, and follow that grid location as the grid evolves through the figure frames. We have only considered $v_{\text{init}} < 0.8$, and we have taken the initial scale hierarchy to be $\kappa = 70 = \ln(m_h/H)$, corresponding to $m_h \sim 10^{11}\text{GeV}$ and $H \sim T^2/m_{pl} \sim 10^{-19}\text{GeV}$ as motivated by an axionic string network at the scale where the network breaks up. We have followed the evolution to $\kappa = 5$, that is, where the string separation is about 100 times larger than the string core size, which is where numerical lattice studies can take over. These choices are not essential; the main conclusion is that ϕ rotates to be very nearly antiparallel, and v grows until the $(1 - v^2)^2$ term in Eq. (3.10) slows down its evolution.

4 Microscopic study of intercommutation

{sec:micro}

The most important result of the last section is that, for virtually all initial string velocities and angles, the microscopic encounter occurs at very high velocity and $\phi \simeq \pi$. Specifically, Figure 3 shows that, for $v_{\text{init}} < 0.6$, the final velocity is in the range $v \in [0.887, 0.924]$ and the angle is in the range $(\pi - \phi) < 0.18$. Therefore the only case of physical relevance is nearly antiparallel strings approaching each other at high velocity $v \simeq 0.9$.

Previous numerical studies [24, 28] have found that global strings approaching each other under these conditions pass through each other without reconnection. We will revisit this conclusion and find that it arose from considering insufficiently large boxes – the parts of the string pair approaching each other at impact parameter $m_h b \in [20, 70]$ are essential to see that interconnection does in fact occur.

In studying the string collision problem, we will use a simplifying approximation, already considered by previous workers [24, 28]. Consider again two strings approaching each other; take the strings to approach along the z axis and to stretch primarily along the y axis, with each string's x position determined by $x = \pm y \tan(\phi/2)$. If the strings are exactly parallel, then the problem reduces to a 2-dimensional problem in which we ignore the y coordinate. If instead the strings are only very nearly parallel, we can make a similar simplification, up to small corrections. The equation of motion for the scalar φ field, derived from Eq. (2.1), is

$$\partial_t^2 \varphi = \partial_x^2 \varphi + \partial_z^2 \varphi + \partial_y^2 \varphi - V'(\varphi). \quad (4.1) \quad \{\text{eq:EOM}\}$$

The field is expected to vary slowly in the y direction, in which case we can drop the $\partial_y^2 \varphi$ term here, with corrections of order $(\tan \phi)^2 \ll 1$. Then for each value of y , the problem becomes a scattering problem of vortices in 2+1 dimensions, with impact parameter b varying along the y -axis as $b = 2x(y) = 2y \tan(\phi/2) \simeq y\phi$. This approximation can break down at large times for some y values. Specifically, defining the location of the vortex in this 2+1D problem to be $(x_v(b, t), z_v(b, t))$, then our approximation has broken down if $dx_v/db > 1/\phi$ or $dz_v/db > 1/\phi$. This will eventually happen, but only at large enough t that interconnection has already happened.

We have studied this problem by implementing Eq. (4.1) (without the $\partial_y^2 \varphi$ term) on a 2+1D lattice. We use a next-nearest-neighbor improved gradient implementation, and a square box which is $L \geq 400/m_h$ on a side. Our initial conditions have two strings, moving with velocities $v = \pm v_0 \hat{z}$ and located at $(x, z) = ((L \pm b)/2, (2 \mp 1)L/4)$. That is, they are separated by the impact parameter b in the x direction and by $L/2$ in the z direction. Our starting condition is that φ/f_a is the complex-number product of the solution of each moving string, and our boundary condition is that we continuously enforce this condition within 2 lattice units of the boundary. This choice avoids any unphysical forces from mirror-image charges, but it means that our simulation can not be trusted and must be stopped after information about the real trajectory of a string has had time to propagate to the boundary and back to the string, roughly $t = L/2$ time. This is why we start the strings some distance off the boundary. For $v = 0.9$ we use a lattice spacing of $m_h a = 0.5$, confirming on $m_h a = 0.33$ lattices that the results are not sensitive to the

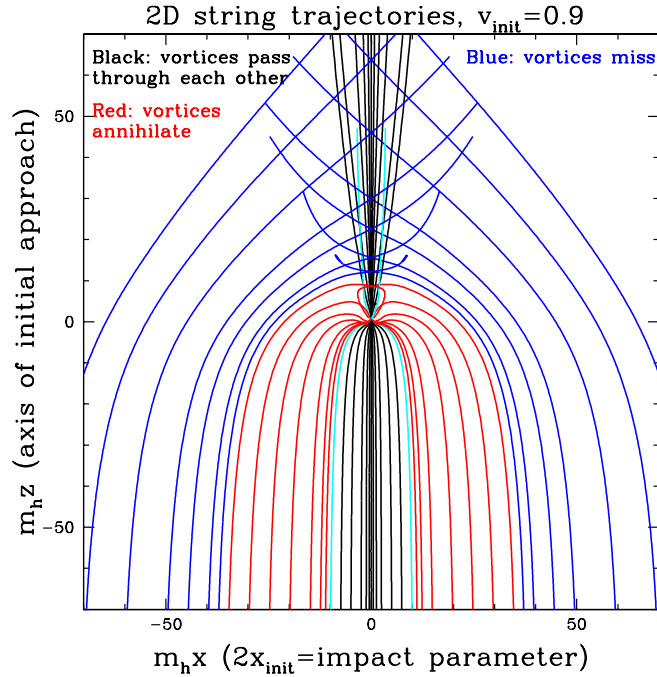


Figure 4. `fig:traject` Each line represents the trajectory of a vortex, approaching an antivortex at $v = 0.9$; each trajectory starts with a different impact parameter. For small impact parameters (black lines) the vortex passes through the antivortex and keeps moving. For intermediate values (red lines) the vortex and antivortex attract each other, collide, and annihilate. For large impact parameters (blue), they miss and keep moving, with a reduced velocity.

spacing. We also study $v = 0.95$ and $v = 0.975$ with $m_h a = 0.25$ and $m_h a = 0.125$ lattices respectively.

We find, in agreement with previous studies, that a vortex-antivortex pair approaching each other with $v = 0.9$ at initial separation $\Delta z = 200/m_h$ and zero impact parameter pass straight through each other with only a small reduction in velocity. But for the strings as a whole to pass through each other, strings must survive passing each other at every impact parameter. Instead, we find for $v = 0.9$ that there is a range of impact parameters, $m_h b \in [21, 70]$, over which the vortex and antivortex curve towards each other and annihilate. For larger b the vortex and antivortex bend as they swing past each other and radiate significant energy, losing much of their velocity; but they escape. We illustrate this behavior in Figure 4 by plotting the trajectory of the upwards-moving vortex (the lower-moving antivortex is an inversion image through the central point) for a number of initial impact parameters. Each curve in the figure represents the trajectory (time-history) of a vortex, entering the plotted region from the bottom as an antivortex enters from the top. For small impact parameter (black lines), the vortices pass through each other at the origin and continue. The cyan trajectory is the last with this property; it slows to a stop at the end of the simulation and will fall back down and annihilate if we follow the simulation longer.

For the next-larger range of impact parameters (red lines), the strings curve to hit each

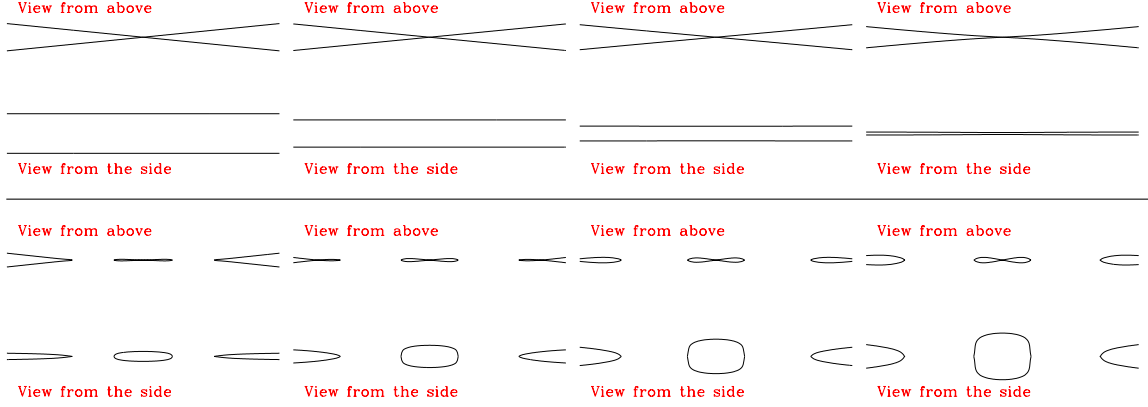


Figure 5. ^{fig:frames} Several single-time frames in a movie of string collision at large velocity and nearly antiparallel orientation, made by solving the 2+1D problem independently in each (x, z) plane. In the first 4 frames, the strings move towards each other. In the fifth frame (leftmost on the second row), they have just crossed. While the strings pass through each other at the center, there is a region to either side where they annihilate, ensuring intercommutation.

other and annihilate. For still larger impact parameters (blue lines), the trajectories curve but miss. In this range, the smaller the impact parameter the more energy is radiated and the slower the string emerges. These strings will also eventually attract each other and collide, but not in the time range available in our study.

We can sew these trajectories together into a string collision picture using the prescription described around Eq. (4.1). We solve the 2+1D vortex-scattering problem for a range of impact parameters, and we compile the solution for each impact parameter at one fixed time to give a single-time “frame” in a movie of the interaction of a pair of strings. We show several equal-time “movie frames” in Figure 5. In other words, we determine the string locations in the (x, z) plane at time t and at a given y by evolving a vortex-antivortex pair with impact parameter $b = 2y \tan(\phi/2)$ for a time t , and then connect together these points at different y to find the string. In making the figure we chose $\phi = 1/5$; making a different choice ϕ_{new} corresponds to stretching the long axis by a factor of $\phi_{\text{old}}/\phi_{\text{new}}$. At or near the point where the strings meet, they pass through each other. On either side of this region there is a region where they annihilate. Far from the intersection point, the strings miss each other. The result is a loop of string at the center, separated by gaps, with the ends reconnected. We include the full movie of the collision in the extra materials accompanying this paper.

In the last three frames of Figure 5, the string loop in the middle has portions where the assumption of smooth y -variation is violated. Therefore the evolution of this loop is not well described. This occurs because the vortex evolution is very sensitive to the impact parameter right at the critical value where the strings either pass through each other or annihilate (around $b = 20/m_h$). But the region where the strings annihilate is wide enough that this cannot causally affect the fact that the central loop is separated from the string on the left and right. Therefore the result that the strings annihilate on either side of this loop is robust.

We repeated the analysis leading to Figure 4 for $v = 0.80$, for $v = 0.95$, and for $v = 0.975$ (using $am_h = 0.25$ for $v = 0.95$ and $am_h = 0.125$ for $v = 0.975$), and found that the same qualitative behavior occurs, but with a slightly different range of impact parameters where string annihilation occurs. For $v = 0.8$ it occurred in $m_h b \in [24, 93]$, for $v = 0.95$ it occurred in $m_h b \in [19, 62]$, and for $v = 0.975$ it occurred in $m_h b \in [20.5, 56]$. Our criterion was that the strings had annihilated and were absent at a time $t = 100/m_h$ after the expected intersection time. Therefore, strings colliding at a (microscopically) nearly antiparallel relative angle will intercommute for all velocities $v < 0.975$ (

5 Discussion and Conclusions

{sec:conclusion}

We have shown that the long-range interactions between global strings play an important role when two strings approach to cross each other. The inter-string forces, operating over a huge logarithmic range of scales, ensure that the microscopic crossing occurs at very high velocity and very nearly antiparallel approach. We also demonstrated, contrary to previous studies, that such high-speed nearly antiparallel string collisions result in intercommutation. Our investigation was based on the fact that, for nearly parallel or antiparallel approach, we can solve the behavior in 2+1 dimensional slices, and that when a 2+1D vortex antivortex pair approach each other relativistically, there is a wide range of nonzero impact parameters over which the strings bend to collide and annihilate. For small impact parameters they pass through each other, but that only causes a small isolated string loop at the string-crossing location; it does not prevent intercommutation.

This result looks peculiar, and it is worth pausing a moment to see if we can understand it. A key feature of those trajectories in Figure 4 which annihilate (the red trajectories in the figure) is that they all have the string's trajectory bend by a large angle before the annihilation. How large a bending angle do we expect? In the approximation that the string's trajectory does not deflect, we find by integrating the second line of Eq. (3.3) over time $\int dt = \int dz/2v$ that the transverse momentum absorbed as the vortices pass each other is $2\pi^2 f_a^2 (1 + v^2)/2v \simeq 2\pi^2 f_a^2$. This is to be compared to the total momentum of the vortex, $\gamma v \pi f_a^2 \ln(bm_h)$. For $\gamma \sim 2$ and moderate impact parameters b , the string is expected to bend by a large angle. A rapid, large deflection by a very relativistic charge results in a large radiated power. The vortex thereby loses a large fraction of its energy. The Coulomb potential is confining in 2+1 dimensions, and the vortex-antivortex pair become tightly bound and fall onto each other at low velocity. And a low-velocity collision does cause annihilation.

This should be the behavior in a range of impact parameters. For larger impact parameter, $\ln(bm_h)$ is larger and the string has more inertia to absorb the bending force. In this case the energy radiated is smaller and the strings do not become tightly bound; instead the string escapes (at least for long time scales). On the other hand, for very small impact parameter, a small bend is enough for the vortices to meet head-on. Little energy is radiated before the collision, and so the strings meet at high velocity and have enough energy to pass through each other.

Our study shows that strings should intercommute if their macroscopic relative velocity is below $v = 0.8$. We expect this to cover the vast majority of string collisions in a network evolution (recall that this is the relative velocity; to achieve it each string must move at $v > 0.8$ straight at the other string). Nevertheless, collisions may occasionally occur with larger velocity. In this case it is no longer true that the microscopic collision angle is small, see Figure 3. Our analysis is not valid here, and it is not clear to us what happens in this case. But previous work [24] suggests that intercommutation should also occur in this case.

Also note that we have only considered one model with global strings, the complex scalar or $O(2)$ (or relativistic xy) model. Models with more complicated vacuum manifolds may show different behavior and would require a separate analysis. This will be needed if any such model proves to be of sufficient physical interest.

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